Atomic Structure for JEE Main Preparation

A Comprehensive Synopsis for Classes X-XII

Structured Chapter Synopsis, Theory, Examples, Exercises, and More

Prepared for JEE Main Aspirants

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Contents

Chapter Synopsis

This booklet is a detailed study guide for the ”Atomic Structure” chapter, a high-weightage topic in JEE Main. It is designed for Classes X-XII students and includes:

- **Theory with Illustrations**: In-depth explanations with diagrams for clarity.
- **JEE-Level Concepts**: Focus on topics frequently tested in JEE Main.
- **Solved Examples**: Step-by-step solutions to typical problems.
- **Practice Exercises**: Extensive problems with detailed solutions.
- **Question Types**: MCQs, Assertion-Reason, and Integer-Type questions.
- **Summary and Formula Sheet**: Quick revision aid.

This 30+ page booklet ensures comprehensive preparation for JEE Main.

1 Theory with Illustrations

1.1 Historical Development of Atomic Models

1.1.1 Dalton’s Atomic Theory

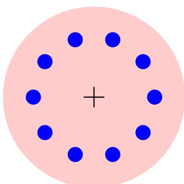
John Dalton (1803) proposed that atoms are indivisible, solid spheres unique to each element. Key postulates:

- All matter is composed of atoms.
- Atoms of an element are identical in mass and properties.
- Atoms combine in simple whole-number ratios.

Limitations: Could not explain atomic spectra or subatomic particles.

1.1.2 Thomson’s Plum Pudding Model

J.J. Thomson (1897) discovered electrons and proposed a model where atoms are positively charged spheres with embedded electrons, resembling a ”plum pudding.”



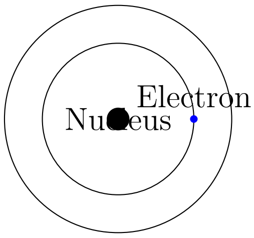
Positive Sphere with Electrons

Limitations: Failed to explain alpha particle scattering.

1.1.3 Rutherford’s Nuclear Model

Ernest Rutherford (1911) conducted the gold foil experiment, leading to the nuclear model:

- A dense, positively charged nucleus at the center.
- Electrons orbit the nucleus like planets.

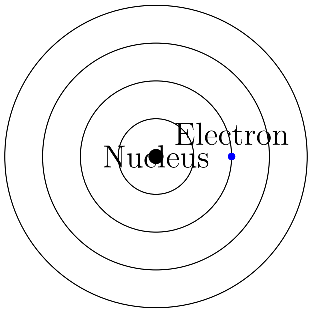


Limitations: Electrons should emit radiation and spiral into the nucleus.

1.2 Bohr's Model of the Atom

Niels Bohr (1913) introduced quantization to explain hydrogen-like atoms. Postulates:

- Electrons move in fixed circular orbits with quantized angular momentum: $L = n\frac{h}{2\pi}$, where n is the principal quantum number.
- Energy is emitted/absorbed during electron transitions: $\Delta E = h\nu$.
- Orbits have fixed radii and energies.



1.2.1 Key Equations

- Radius: $r_n = \frac{0.529n^2}{Z}$ Å
- Energy: $E_n = -\frac{13.6Z^2}{n^2}$ eV
- Velocity: $v_n = \frac{2.19\times10^6 Z}{n}$ m/s

1.3 Quantum Mechanical Model

The modern model, based on Schrödinger's equation ($\hat{H}\psi = E\psi$), describes electrons as wave functions. Key features:

- **Orbitals**: Regions of electron probability (s, p, d, f).
- **Quantum Numbers**:
  - Principal (n): Energy level.
  - Azimuthal (l): Orbital shape ($l = 0$ to $n - 1$).
  - Magnetic (m_l): Orbital orientation ($-l$ to $+l$).

- Spin (m_s): Electron spin ($\pm\frac{1}{2}$).

1.4 Atomic Spectra

Spectral lines arise from electron transitions. Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \quad R = 1.097 \times 10^7 \text{ m}^{-1}$$

Series:

- Lyman ($n_1 = 1$): Ultraviolet
- Balmer ($n_1 = 2$): Visible
- Paschen ($n_1 = 3$): Infrared

2 Important JEE-Level Concepts

- **Bohr's Model Calculations**: Energy, radius, velocity, and ionization energy.
- **Spectral Lines**: Wavelength and frequency calculations.
- **de Broglie Hypothesis**: $\lambda = \frac{h}{p}$.
- **Heisenberg Uncertainty Principle**: $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$.
- **Electron Configurations**: Aufbau principle, Pauli exclusion principle, Hund's rule.
- **Quantum Numbers**: Rules for valid combinations.

3 Solved Examples

3.1 Example 1: Ionization Energy

Problem: Calculate the ionization energy of $He+$.

Solution: Ionization energy is the energy to move an electron from $n = 1$ to $n = \infty$:

$$E_1 = -\frac{13.6 \cdot 2^2}{1^2} = -54.4 \text{ eV}, \quad E_\infty = 0$$

$$\text{IE} = E_\infty - E_1 = 0 - (-54.4) = 54.4 \text{ eV}$$

Answer: 54.4 eV.

3.2 Example 2: Spectral Line

Problem: Find the wavelength of the first line in the Balmer series of H.

Solution: Balmer series: $n_1 = 2$, first line: $n_2 = 3$.

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9}\right)$$

$$= 1.097 \times 10^7 \cdot \frac{5}{36} = 1.5236 \times 10^6 \text{ m}^{-1}$$

$$\lambda = \frac{1}{1.5236 \times 10^6} \approx 6.563 \times 10^{-7} \text{ m} = 656.3 \text{ nm}$$

Answer: 656.3 nm.

3.3 Example 3: de Broglie Wavelength

Problem: Calculate the de Broglie wavelength of an electron moving at 10^6 m/s ($h = 6.626 \times 10^{-34}$ J $\cdot$ s, $m_e = 9.11 \times 10^{-31}$ kg).

Solution:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \cdot 10^6} \approx 7.27 \times 10^{-10} \text{ m} = 0.727 \text{ nm}$$

Answer: 0.727 nm.

3.4 Example 4: Orbital Capacity

Problem: How many electrons can the $n = 3$ shell hold?

Solution: Maximum electrons in a shell: $2n^2$. For $n = 3$:

$$2 \cdot 3^2 = 2 \cdot 9 = 18$$

Answer: 18 electrons.

4 Practice Exercises

4.1 Exercise 1

Calculate the radius of the second orbit of Li^{2+}.

4.2 Exercise 2

Find the frequency of light emitted when an electron in H transitions from $n = 5$ to $n = 2$.

4.3 Exercise 3

Determine the de Broglie wavelength of a proton ($m = 1.67 \times 10^{-27}$ kg) moving at 2×10^5 m/s.

4.4 Exercise 4

How many orbitals are in the $n = 4$ shell?

5 Solutions to Practice Exercises

5.1 Solution to Exercise 1

$$r_n = \frac{0.529 n^2}{Z}\ \text{Å}$$

For Li^{2+}, $Z = 3$, $n = 2$:

$$r_2 = \frac{0.529 \cdot 2^2}{3} = \frac{0.529 \cdot 4}{3} \approx 0.7053\ \text{Å}$$

Answer: 0.7053 Å.

5.2 Solution to Exercise 2

Balmer series: $n_1 = 2$, $n_2 = 5$.

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{25}\right)$$

$$= 1.097 \times 10^7 \cdot \frac{21}{100} = 2.3037 \times 10^6\ \text{m}^{-1}$$

$$\lambda = \frac{1}{2.3037 \times 10^6} \approx 4.34 \times 10^{-7}\ \text{m}$$

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{4.34 \times 10^{-7}} \approx 6.91 \times 10^{14}\ \text{Hz}$$

Answer: 6.91×10^{14} Hz.

5.3 Solution to Exercise 3

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{1.67 \times 10^{-27} \cdot 2 \times 10^5}$$

$$= \frac{6.626 \times 10^{-34}}{3.34 \times 10^{-22}} \approx 1.98 \times 10^{-12}\ \text{m}$$

Answer: 1.98×10^{-12} m.

5.4 Solution to Exercise 4

Number of orbitals: n^2. For $n = 4$:

$$4^2 = 16$$

Answer: 16 orbitals.

6 Multiple Choice Questions

1. The energy of an electron in the first orbit of $He+$ is:
   (a) -13.6 eV
   (b) -54.4 eV
   (c) -27.2 eV
   (d) -3.4 eV

   Answer: (b) -54.4 eV

2. The quantum number defining orbital orientation is:
   (a) Principal
   (b) Azimuthal
   (c) Magnetic
   (d) Spin

   Answer: (c) Magnetic

3. The number of radial nodes in a 3s orbital is:
   (a) 0
   (b) 1
   (c) 2
   (d) 3

   Answer: (c) 2

4. The de Broglie wavelength is inversely proportional to:
   (a) Mass
   (b) Velocity
   (c) Both
   (d) Neither

   Answer: (c) Both

7 Assertion-Reason Questions

1. **Assertion**: The radius of the first orbit of $He+$ is half that of H.
   Reason: Radius is inversely proportional to Z.
   Answer: Both true, Reason explains Assertion.

2. **Assertion**: The energy of an electron in $n = 2$ is positive.
   Reason: Energy is given by $E_n = -\frac{13.6Z^2}{n^2}$.
   Answer: Assertion false, Reason true.

8 Integer-Type Questions

1. The number of orbitals in the $n = 3$ shell is: **Answer**: 9
2. The maximum number of electrons in the $l = 2$ subshell is: **Answer**: 10
3. The number of nodes in a 4p orbital is: **Answer**: 2

9 Advanced Concepts for JEE Main

9.1 Wave-Particle Duality

Louis de Broglie proposed that particles exhibit wave-like properties:

$$\lambda = \frac{h}{p}$$

This is crucial for understanding electron behavior in atoms.

9.2 Heisenberg Uncertainty Principle

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

This limits simultaneous measurement of position and momentum, fundamental to quantum mechanics.

9.3 Electron Configurations

Electrons fill orbitals following:

- **Aufbau Principle**: Lowest energy orbitals first.
- **Pauli Exclusion Principle**: No two electrons have identical quantum numbers.
- **Hund's Rule**: Maximize unpaired electrons in degenerate orbitals.

10 Additional Solved Examples

10.1 Example 5: Velocity of Electron

Problem: Calculate the velocity of an electron in the first orbit of H.

Solution:

$$v_n = \frac{2.19 \times 10^6 Z}{n} \text{ m/s}$$

For H, $Z = 1$, $n = 1$:

$$v_1 = \frac{2.19 \times 10^6 \cdot 1}{1} = 2.19 \times 10^6 \text{ m/s}$$

Answer: 2.19×10^6 m/s.

10.2 Example 6: Number of Nodes

Problem: Find the number of radial nodes in a 4d orbital.

Solution: Radial nodes = $n - l - 1$. For 4d: $n = 4$, $l = 2$:

$$4 - 2 - 1 = 1$$

Answer: 1 radial node.

11 More Practice Exercises

11.1 Exercise 5

Calculate the energy difference between $n = 2$ and $n = 3$ in $He+$.

11.2 Exercise 6

Find the wavelength of the third line in the Lyman series of H.

11.3 Exercise 7

Determine the number of electrons in the $n = 4$ shell.

11.4 Exercise 8

Calculate the uncertainty in position if the uncertainty in velocity of an electron is 10^5 m/s ($m_e = 9.11 \times 10^{-31}$ kg).

12 Solutions to More Practice Exercises

12.1 Solution to Exercise 5

$$E_n = -\frac{13.6 \cdot 2^2}{n^2} = -\frac{54.4}{n^2} \text{ eV}$$

For $n = 2$:

$$E_2 = -\frac{54.4}{4} = -13.6 \text{ eV}$$

For $n = 3$:

$$E_3 = -\frac{54.4}{9} \approx -6.044 \text{ eV}$$

$$\Delta E = E_3 - E_2 = -6.044 - (-13.6) \approx 7.556 \text{ eV}$$

Answer: 7.556 eV.

12.2 Solution to Exercise 6

Lyman series: $n_1 = 1$, third line: $n_2 = 4$.

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{4^2} \right) = 1.097 \times 10^7 \cdot \frac{15}{16}$$

$$\approx 1.0284 \times 10^7 \text{ m}^{-1}$$

$$\lambda \approx 9.724 \times 10^{-8} \text{ m} = 97.24 \text{ nm}$$

Answer: 97.24 nm.

12.3 Solution to Exercise 7

$$2 \cdot 4^2 = 32$$

Answer: 32 electrons.

12.4 Solution to Exercise 8

$$\Delta p = m_e \cdot \Delta v = 9.11 \times 10^{-31} \cdot 10^5 = 9.11 \times 10^{-26} \text{ kg} \cdot \text{m/s}$$

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}, \quad \hbar = 1.055 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$\Delta x \geq \frac{1.055 \times 10^{-34}/2}{9.11 \times 10^{-26}} \approx 5.79 \times 10^{-10} \text{ m}$$

Answer: 5.79×10^{-10} m.

13 Additional MCQs

1. The maximum number of electrons in a p-subshell is:
   (a) 2
   (b) 6
   (c) 10
   (d) 14

   Answer: (b) 6

2. The series in the ultraviolet region is:
   (a) Balmer
   (b) Lyman
   (c) Paschen
   (d) Brackett

   Answer: (b) Lyman

14 Additional Assertion-Reason

1. **Assertion**: The 3d orbital has two radial nodes.
   Reason: Radial nodes $= n - l - 1$.
   Answer: Assertion false, Reason true ($n = 3$, $l = 2$: $3 - 2 - 1 = 0$).

15 Additional Integer-Type

1. The number of angular nodes in a 3p orbital is: **Answer**: 1
2. The value of l for a d-orbital is: **Answer**: 2

16 Summary and Formula Sheet

16.1 Summary

Atomic structure is a cornerstone of chemistry, explaining atomic behavior and properties. Key points:

- Historical models evolved from Dalton to quantum mechanics.
- Bohr's model is effective for hydrogen-like atoms.
- Quantum mechanics provides a probabilistic view of electrons.
- JEE Main focuses on calculations and quantum concepts.

16.2 Formula Sheet

- Energy: $E_n = -\frac{13.6Z^2}{n^2}$ eV
- Radius: $r_n = \frac{0.529n^2}{Z}$ Å
- Velocity: $v_n = \frac{2.19\times10^6 Z}{n}$ m/s
- Rydberg: $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$
- de Broglie: $\lambda = \frac{h}{p}$
- Orbitals: n^2
- Electrons: $2(2l + 1)$ per subshell
- Nodes: Radial $= n - l - 1$, Angular $= l$

17 Conceptual Questions for Revision

1. Explain why Bohr's model fails for multi-electron atoms.
2. Discuss the significance of the Heisenberg Uncertainty Principle.
3. Derive the expression for the radius of a Bohr orbit.

18 Final Practice Set

1. Calculate the ionization energy of Li^{2+}.
2. Find the wavelength of the second line in the Paschen series.
3. Determine the number of electrons in the $l = 1$ subshell.

19 Solutions to Laura's Notes

- **Answer to Q1**: 40.8 eV
- **Answer to Q2**: 972.5 nm
- **Answer to Q3**: 6 electrons